W4L1 - INTRO TO LAPLACE TRANSFORMS

The Laplace Transform is a method to solve some differential equations.

Definition: Suppose that $f(t)$ is a piecewise continuous function for $t \geq 0$. The Laplace transform of $f(t)$ is:

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st}\, dt \quad , \; s > 0$$

provided the integral converges.

Common notation:

$$\mathcal{L}[f(t)] = F(s) \quad \text{or} \quad \mathcal{L}[y(t)] = Y(s)$$

$$\mathcal{L}^{-1}[F(s)] = f(t) \quad \text{or} \quad \mathcal{L}^{-1}[Y(s)] = y(t)$$

Process:

Differential Equation (IVP) ⟶ Use Laplace Transforms to Create an Algebraic Equation ⟶ Solve Algebraic Equation ⟶ Use Inverse Laplace Transforms To Solve Original DE

EX: Given: $y'' - 6y' + 8y = 0$, $y(0) = 2$, $y'(0) = 0$

$y'' - 6y' + 8y = 0$ Start with the given differential equation

$\mathcal{L}[y''] - \mathcal{L}[6y'] + \mathcal{L}[8y] = \mathcal{L}[0]$ Take the Laplace transform of each term

$s^2 Y(s) - s \cdot y(0) - y'(0) + 6(s \cdot Y(s) - y(0)) + 8\,Y(s) = 0$

$Y(s) = \dfrac{-2s - 12}{s^2 + 6s + 8}$ Solve algebraic equation for $Y(s)$

$\mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\dfrac{-2s - 12}{s^2 + 6s + 8}\right]$ Take the inverse Laplace transform of both sides of the equation

$y(t) = 2e^{-4t} - 4e^{-2t}$ Solution